Rr study appendix D

Miscellaneous Bits

Rr reference point

Terman mentions the reference point because the feedpoint is often not coincident with the current maximum ("loop") as indicated in figure 1 (from Johnk[1]).

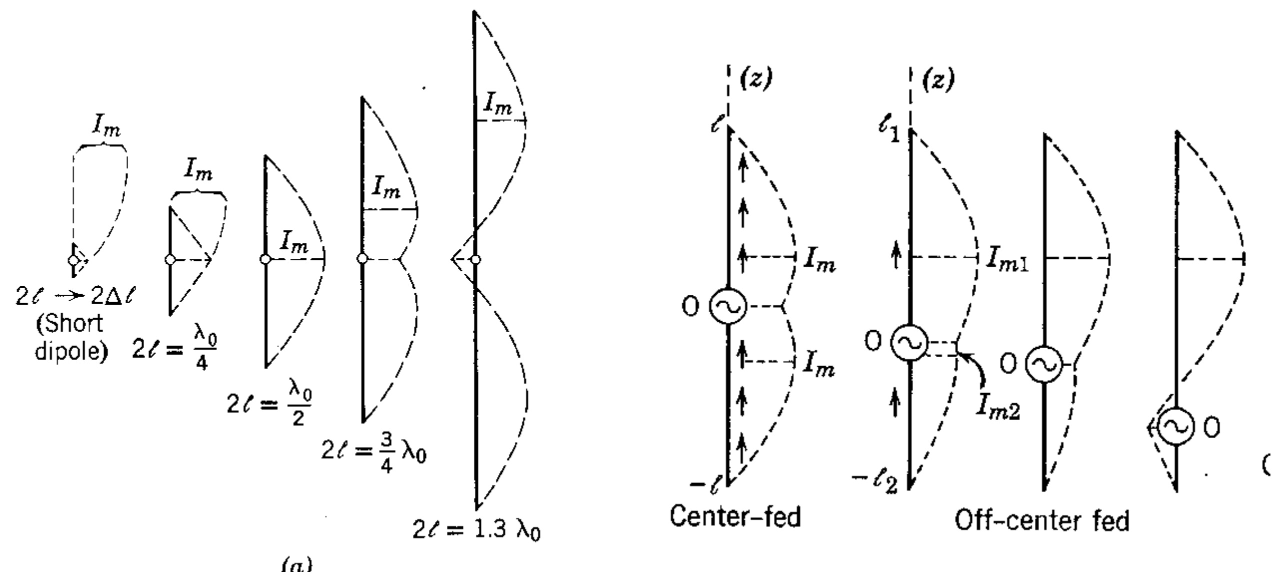

Figure D1 - typical cu**rr**ent distributions.

Some authors (Stutzman & Thiele[2] for example) will identify both possibilities with the notation **Rrm** or **Rri** referencing either the maximum current point or the feedpoint respectively.

<u>Some experimental and modeling data</u>

Assuming losses other than **Rg** are small, the measured **Ri** for a vertical has traditionally been assumed to consist of two parts, **Rr** and **Rg**. Typically for ground mounted verticals **Rr** is assumed to be the value of **Rr** for the antenna over an infinite perfect ground-plane and **Rg = Ri - Rr**. An example of this thinking can be seen in the classic paper by Brown, Lewis and Epstein[3] (BLE).

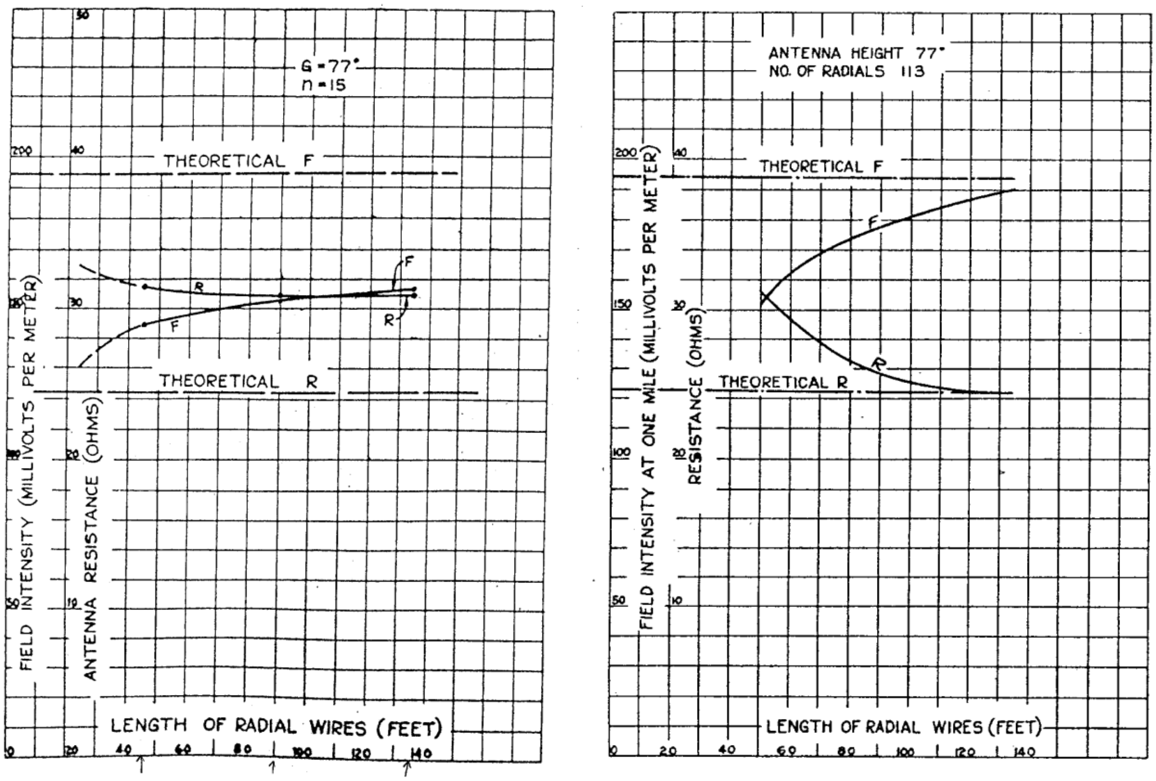

Figure D3 - **Ri** and field strength (**F**) from BLE[3], figures 38 (right) and 39 (left).

Figure D3 shows two graphs taken from their paper. The test antenna had a theoretical **Rr** ≈ 24.5 Ω. The graphs show the measured **Ri** and field strength (**F**) for two numbers of radials (15 and 113) as the radial length was varied. In the case of 15 radials, as the radial length is increased, **Ri** converges on ≈31 Ω, indicating **Rg** ≈6.5Ω. In the case of 113 radials, **Ri** converges on the theoretical value **Rr**≈24.5Ω indicating that when a large number of long radials are used **Rg** becomes very small. Ok, this is what we're accustomed to seeing and it fits our conceptual model in figure 2 where **Rr** is assumed to be the value over perfect ground. However, in addition to **Ri** there is plot of **F** on each graph. In both cases while **Ri** has flattened out at longer radial lengths, **F** continues to rise. Using our conceptual model this implies that the efficiency continues to improve even though **Ri** has stabilized? It would appear that **Rg** continues to decrease and **Rr** increases keeping the sum the same. Does variation in <u>both</u> **Rr** and **Rg** actually explain the apparent contradiction between the two curves? The BLE paper isn't the only place we see this a possible contradiction, I've seen it in other papers and in my own measurements. Some times **Ri** not only flattens out but starts to increase as longer or more numerous radials are employed (see Wait[4]).

Some years ago while modeling 80m verticals I wrote down the following comments (the figure numbers have been modified to fit this article):

"Figure D4 gives an example using NEC4 modeling where we vary the buried radial length at a given frequency.

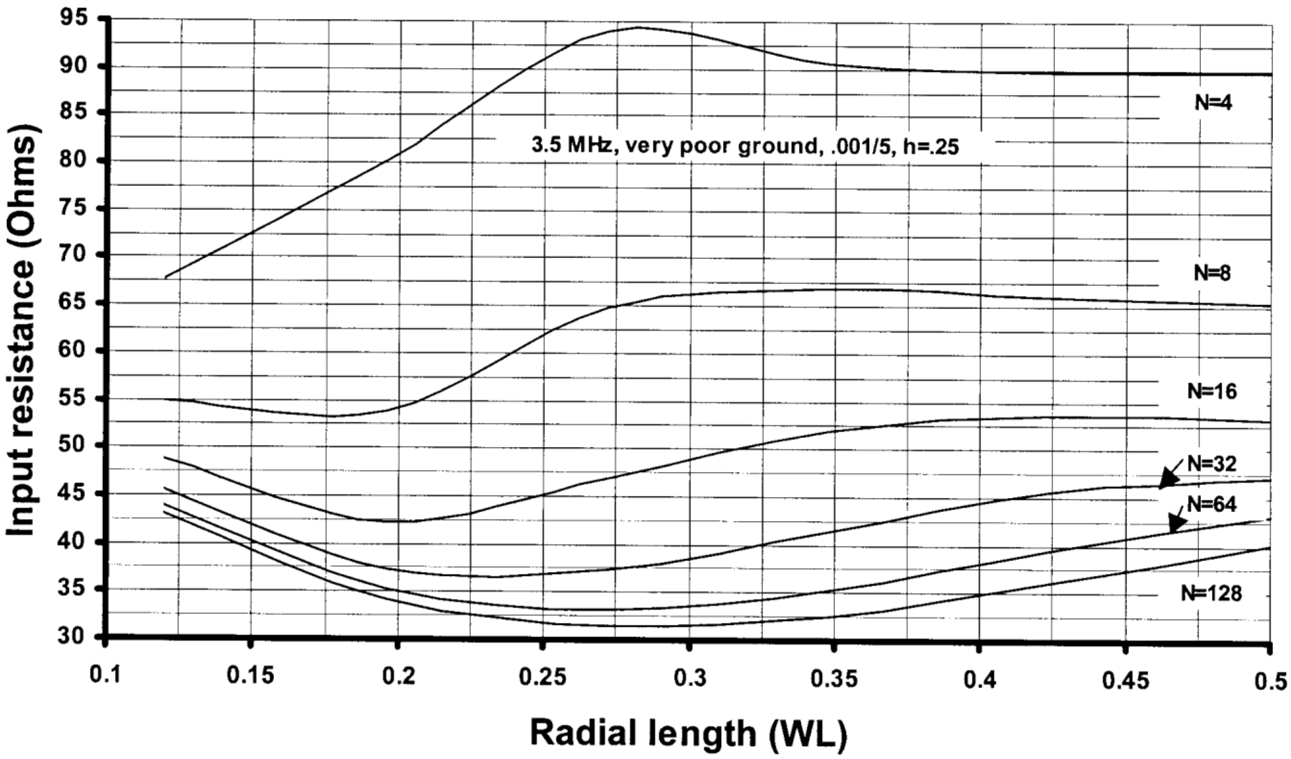

Figure D4, Input resistance from NEC4 modeling

*At first glance this graph is crazy! For example, if we assume that **Rr** = 36.6 Ohm, we see that for the larger values of N, the input resistance (**Ri**) is substantially less than this for some radial lengths, implying negative **Rg**. Notice also that for N=4, lengthening the radials increases **Rg** right off the bat! For larger numbers of radials, initially lengthening the radials does reduce the input resistance but when the radials are long enough, up goes the resistance again. If we interpret the input resistance to consist of **Rr** over ideal ground plus **Rg** due to ground losses then these curves don't make sense.*

*The idea that we can simply subtract the ideal **Rr** from R_i to determine **Rg** may not be correct. The radial system has an effect on **Rr** even when the radials are buried. As we change the number and length of radials, **Rr** oscillates around some value and the range of variation can make **Rg** appear to be lower or higher than it really is. Figure D5 is an example of this oscillation for a 0.25 wl vertical over a perfectly conducting disk of radius "a", in free space, as we vary the radius where k = 2π/λ. ka is 2π times the disk radius in wavelengths. For example, ka=5 is about 0.8λ. This graph is taken from Leitner and Spence [5]. The graph shows both experimental and calculated values.*

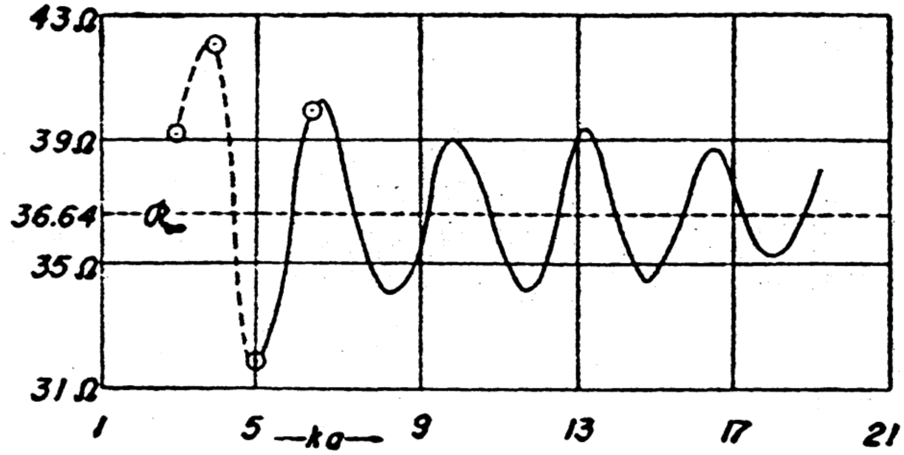

*Figure D5- Variation in **Rr** with disc radius.*

***Rr** oscillates around 36.6 Ω by about $\pm 5\ \Omega$. This is a highly idealized case but when we repeat it using radial wires rather than a conducting disk, the oscillations in **Rr** get even larger, especially for small N. When we immerse the radial system in ground, the oscillations are damped but still present. For poor ground the damping is not all that great as we saw in figure 4. For high conductivity ground the oscillations almost disappear as shown in figure D6."*

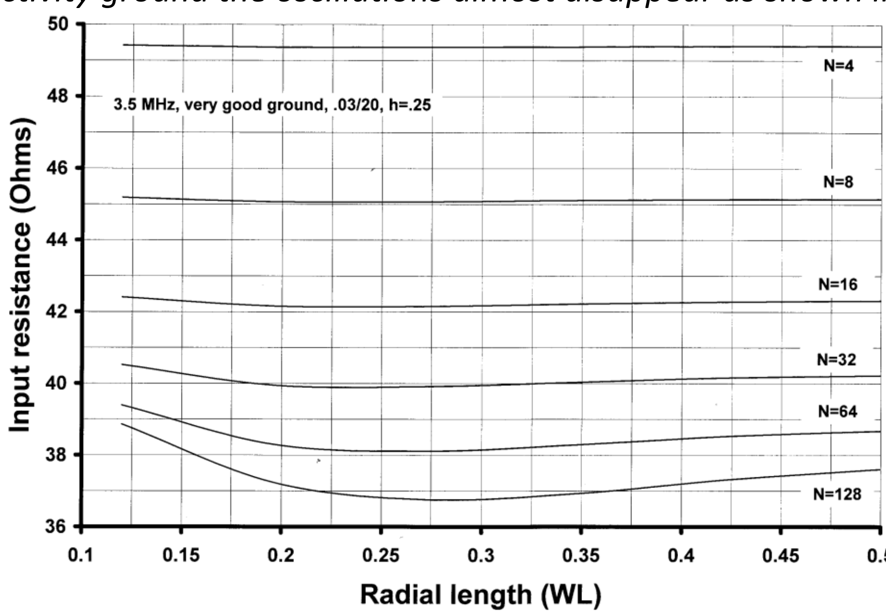

Figure D6, Variation in R_i over very good ground

It turns out that I'm not the first to note this. Wait[4] shows this and in a 1936 IRE paper, Hansen and Beckerley [6] calculated **Rr** directly from the radiated power over real ground. What they show is the effect of ground on **Rr** for a range of ground constants. For perfect ground they get 36.6Ω but for real grounds, values for **Rr** are in the range of 16-25Ω for H=0.25λ. This paper was written by two physicists at Stanford. Although an IRE paper it's highly mathematical and uses Heavyside-Lorentz units rather than MKS. I suspect it reached a very small audience.

For verticals where the height is greater than $\approx\lambda_o/8$ these details are pretty much a matter of academic interest. However, for very short antennas, like those we'll be using on 630m where we're interested in the radiated power for a given input power, the interplay between **Rr** and **Rg** is of practical concern. One example would be recent measurements on my 630m transmitting antenna. The antenna is a 95' vertical with a 240' diameter top-loading hat. Initially I installed sixty four 150' radials around the base and made a measurement of $\mathbf{Z_i=R_i+X_i}$ using an vector network analyzer (VNA). I then added another sixty four 150' radials to bring the total up to 128. With twice as many radials **Ri** increased! By doubling the number of radials I expected I would reduce **Rg** somewhat and that would be reflected in a lower value for **Ri** but instead it went up. I wouldn't expect ground losses to increase with more radials so it appears that there was a small increase in **Rr** which does not agree with conventional thinking!

These examples provided motivation for trying to understand what's going on.

Calculating Rr in short verticals

There are a number of different ways to calculate the perfect ground value for **Rr** in short loaded verticals. I usually use the method outlined by Edmund Laport[7]. Some other approaches use the concept of equivalent height which gives the same results. Some years back I wrote up some comments on Laport's approach. I've folded those notes into this appendix.

In 1954 I purchased "Radio Antenna Engineering", by Edmund Laport[7]. I still have this book although my copy shows it's age having been carried over much of the world and soaked in seawater on occasion. I've always found the book to be very helpful and it's been one of my standard references for antennas. Recently I mentioned this book to Paul Kiciak, N2PK, during a discussion of short loaded vertical antennas. Paul wondered if Laport was really correct with regard to the current distributions shown for short verticals. His questions made me realize that I had accepted the information in the book rather uncritically for the past 60 years. While I understood what his sources were I had not checked the material (in particular the discussion in chapter 1 on low frequency antennas) using NEC to see if it agreed with Laport.

That was easy to remedy! The following is a very short study comparing NEC results to some of the graphs in chapter 1 of Laport. I focused on figures 1.1 and 1.2 which I've used frequently for preliminary designs of short loaded verticals.

Caveats!

The graphs in Laport are based on mathematical approximations using the assumption of sinusoidal (or a portion of a sinusoid) current distribution on the antenna. While not strictly true on real antennas the sinusoidal approximation has been shown to be very good in most situations, especially for antennas where the height (H) is less than λ/2. When modeling with NEC the wires are divided into segments and the currents are given at the center of each segment and assumed constant along a given segment. In this discussion the modeling frequency is 1.83 MHz and I used segment lengths of 1'.

It's well known that due to the diameter of the conductor a wire will be electrically a few percent longer than it is physically. Laport does not take that into account. His heights (H) are the effective electrical height in degrees. NEC on the other hand does take this into account. Laport assumes the top loading is in the form of a disc but for modeling I've used a number of radial wires for the top hat. These effects introduce small differences.

Laport Figure 1.1

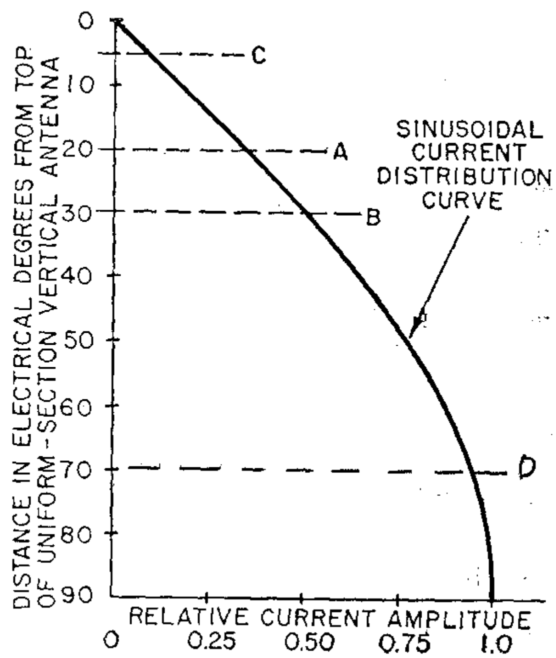

Figure D7 - Figure 1.1 from Laport showing the current distributions on verticals of different heights.

Figure D7 can be used in two ways: to show the current distribution on a unloaded short vertical (<90°), i.e. the current distribution above lines A-D for different heights or to show the current distribution on a short loaded antenna, i.e. the distributions below lines A-D. The distributions below a given line assume that enough top-loading has been used to resonate the antenna at the operating frequency. I modeled both these cases with EZNEC pro v5 using the NEC4D engine. For simplicity I used perfect ground and lossless conductors. All the conductors were #12 wire.

Figure D7 shows the current on the vertical is zero at the top and increases as you proceed downward towards the base. If the antenna is very short the current distribution is essentially linear. Figure D8 compares the modeled current distribution with H = 60° to a sine function. As can be seen, the agreement between NEC and Laport is very good.

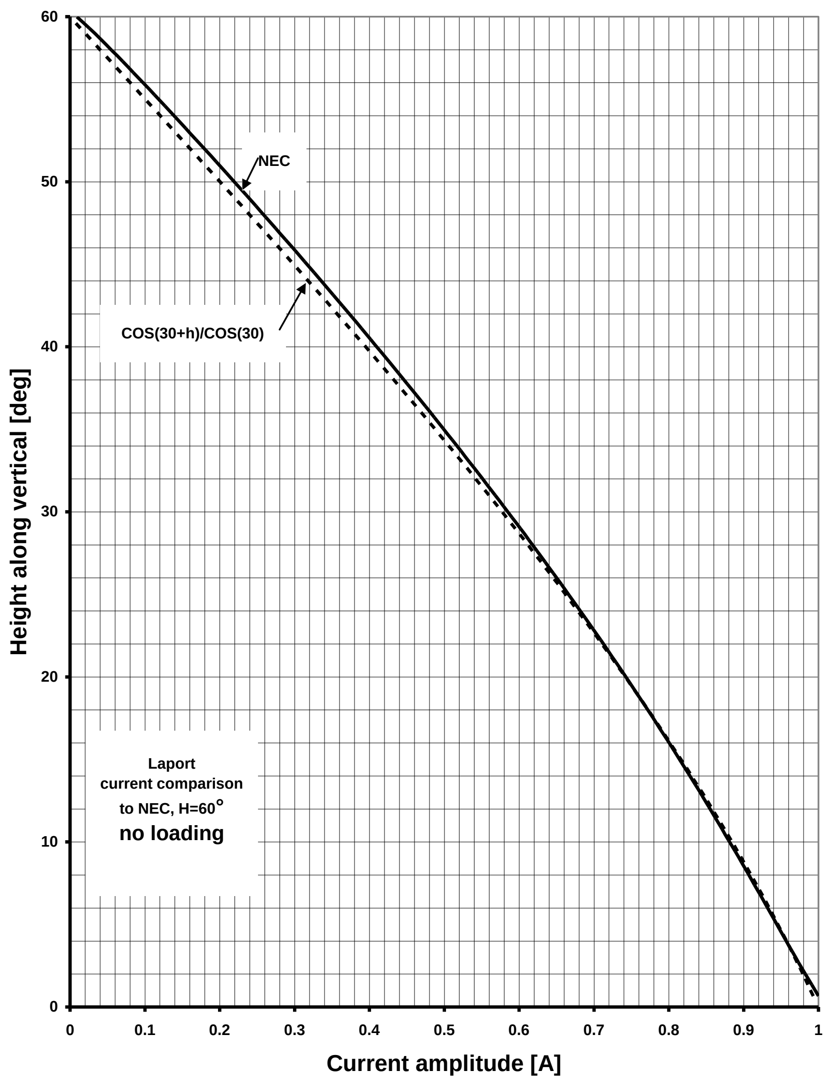

Figure D8 - Comparison of NEC modeling versus Laport for an H=60° unloaded vertical.

Another possibility would be to chop off the top of the antenna and replace it with a capacitive disc or several horizontal radial wires long enough to resonate the antenna. The NEC model I used is shown in figure D9.

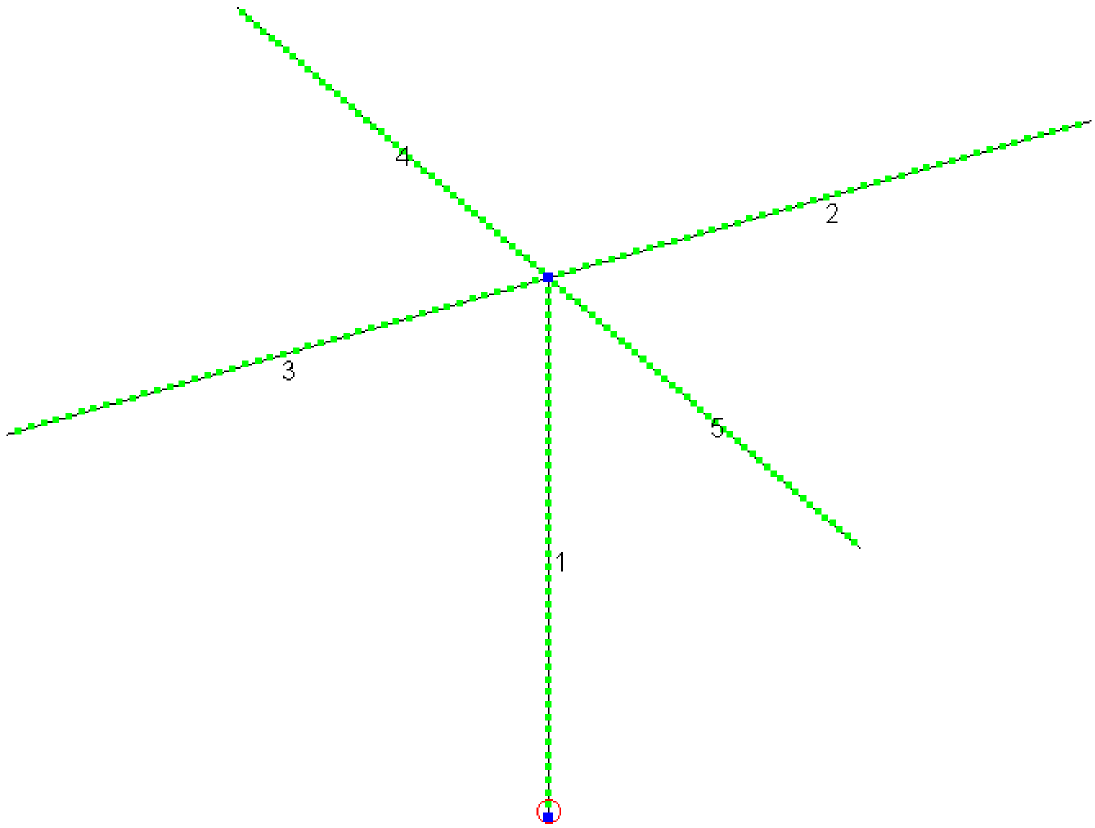

Figure D9 – NEC model for a short (H=30°) top-loaded vertical

Figure D10 compares the current distribution on this model to a cos(h) distribution for H = 30° with two types of top loading to resonate: four radial wires only and a combination of radial wires and an inductor placed right under the top hat. Over most of the vertical we see good agreement between Laport and NEC. However, at the top of the antenna there is a small difference. As we'll see in the next section, this small difference in current distribution doesn't seem to have much effect on the NEC radiation resistance ($\mathbf{R_r}$) values compared to Laport's calculation. In any case Laport's profile is a good approximation.

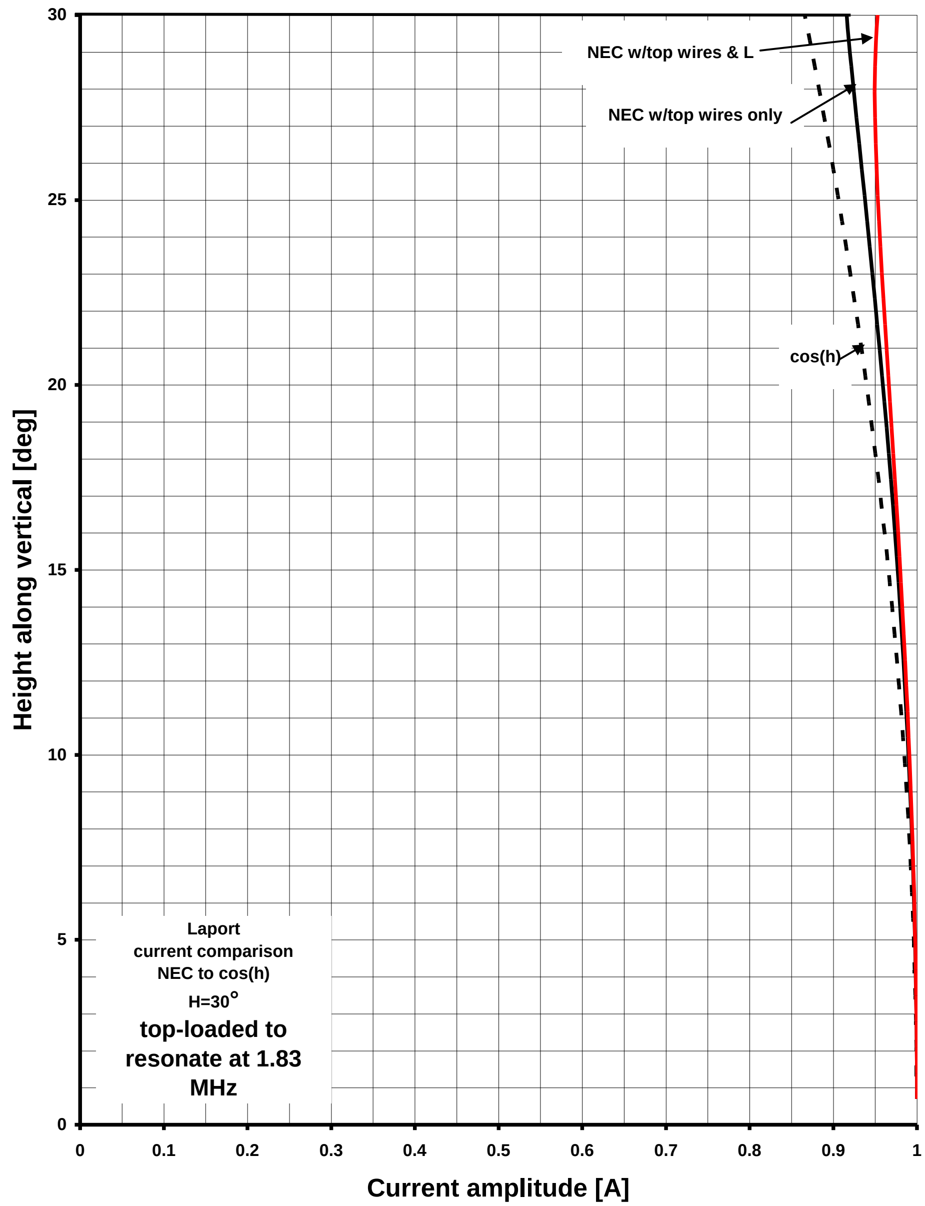

Figure D10 – Comparison between NEC and Laport for H=30° with two forms of top loading:

42’ radials wires alone and 20’ radial wires with an inductor at the top of the vertical wire.

Laport’s figure 1.1 gives a reasonable idea of what to expect for current distribution on at least some types of short verticals.

<u>Laport figure 1.2</u>

In chapter 1 Laport gives a simple approximation for calculating the radiation resistance (**Rr**) of short verticals. He states that this expression is valid for H < 30° but it seems to work well up to H = 50° at least.

The expression he uses is:

$$R_r = 0.01215\, A^2$$
$$Where$$
$$A = \frac{H}{2}\left(\frac{I_{top}}{I_{base}} + 1\right)$$

H is in degrees. I inserted this expression into EXCEL and generated the graph shown in figure D11 which reproduces Laport's figure 1.2.

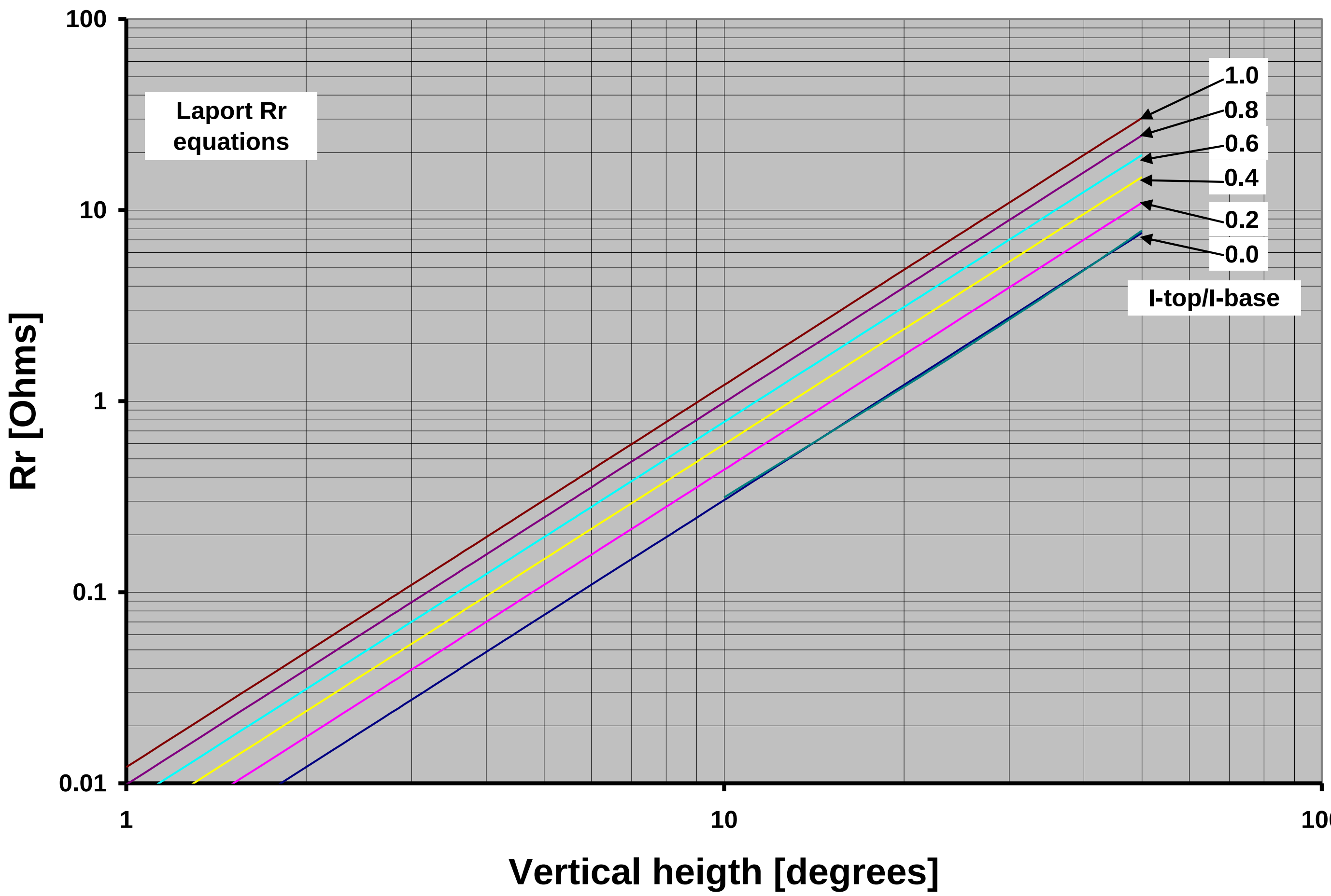

Figure D11 – Radiation resistance, at the base, for short loaded verticals.

To check Laport I modeled several top-loaded verticals with different heights and I_{top}/I_{base} ratios and compared the R_r values between NEC and Laport. Here's what I got:

Table 1

	NEC	Laport	NEC	Laport
	It/Ib=0	It/Ib=0	It/Ib=0.8	It/Ib=0.8
height [degrees]	**Rr**	**Rr**	**Rr**	**Rr**
10	0.31	0.30	0.94	0.98
20	1.19	1.22	3.57	3.94
30	2.67	2.73	8.05	8.86
40	4.83	4.86	14.45	15.75
50	7.79	7.59	23.08	24.60

As can be seen, the agreement is pretty good. More than adequate for an initial design.

Laport's expression relates $\mathbf{R_r}$ to the Ampere-degree area of the current distribution (i.e. the integral of the current over H in degrees). In the ARRL Antenna Compendium, Volume 1, 1985, pp. 108-115, Bruce Brown, W6TWW (sk), wrote an article entitled "Optimum Design of Short Coil-Loaded High-Frequency Mobile Antennas. What Bruce did was to extend Laport's concept of "Ampere-degree area" to verticals without top loading but with the coil inserted part way up the vertical. There has been some discussion on his treatment of the current distribution across the loading coil. The current profile across the coil is not constant as he assumes but decreases somewhat from bottom to top. The magnitude of difference in current amplitude between the ends of the loading coil is however, a matter of some dispute but I still feel Bruce's work has considerable merit.

Conclusion

I think Laport's work agrees well with NEC and has the advantage that for a given antenna height you can get a good idea of the current distribution and the radiation resistance by inspection with only a few minutes of effort. That's a great starting point for the design of a short antenna where the next step is to NEC modeling.

ERP, EIRP and radiated power

On 630m the maximum allowable power is expected to be stated in terms of "effective isotropic radiated power" (EIRP) which is not the same as the total radiated power, Pr. The radiated power is the actual power radiated by the antenna: Pr=Rr Io^2, Io in Arms. The allowable power can also be expressed in terms of the "effective radiated power" (ERP) which is not the same as EIRP! It's important to understand the differences. Our interest is to determine the allowable Pr for a given ERP or EIRP limit for the short verticals likely to be used by amateurs on 630m.

EIRP is referenced to an "isotropic" radiator in free space. An isotropic radiator radiates uniformly in all directions, i.e. if you measure the power density (Pdi, in W/m^2) on the surface of a hypothetical sphere surrounding an isotropic radiator you'll find Pdi is the same everywhere. ERP is different, it is referenced to the power radiated by a λ/2 dipole in free space.

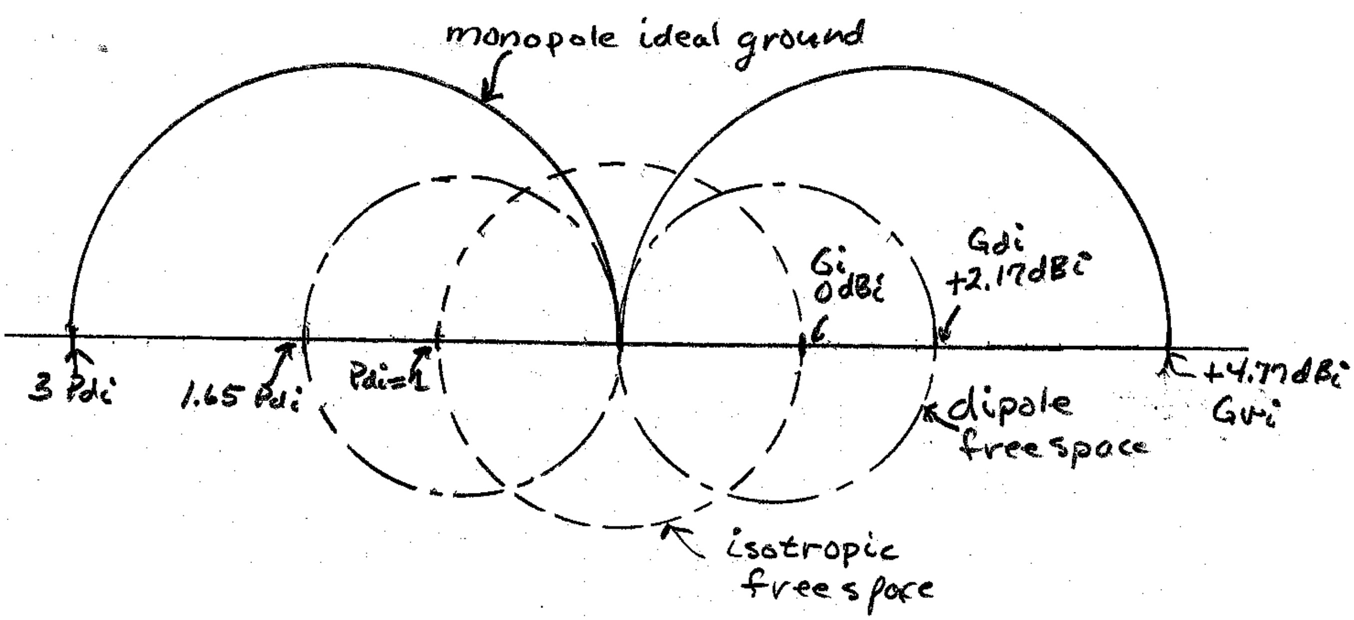

Figure SB1 - Radiation power density at the same radius from an isotropic radiator in free space, a λ/2 dipole in free space and a short monopole over perfect ground.

Figure SB1 compares the radiation patterns of an isotropic radiator in free space, a dipole in free space and a short vertical over ideal ground. Gi = 1 (0 dBi) is the gain of the isotropic radiator. For the dipole the gain relative to the isotropic is Gdi= 1.648 (+2.17 dBi). For the short vertical over ideal ground the gain relative to the isotropic is Gvi=3 (+4.77 dBi). When you place a short monopole over a perfect ground-plane, for the same Pr, the power density at the same radius will be greater by a factor of 3 (+4.77 dB) because the power is being radiated into a hemisphere rather than a sphere because of reflection from the ideal ground doubles Pd and there is a further increase of 1.5X (+1.77 dB) due to the directivity of the short monopole.

The gain of the vertical when referenced to the dipole is Gvd= (4.77 dBi - 2.17 dBi) = 2.6 dBi or a factor of 1.82. Figure SB2 shows these relationships in a flowchart format.

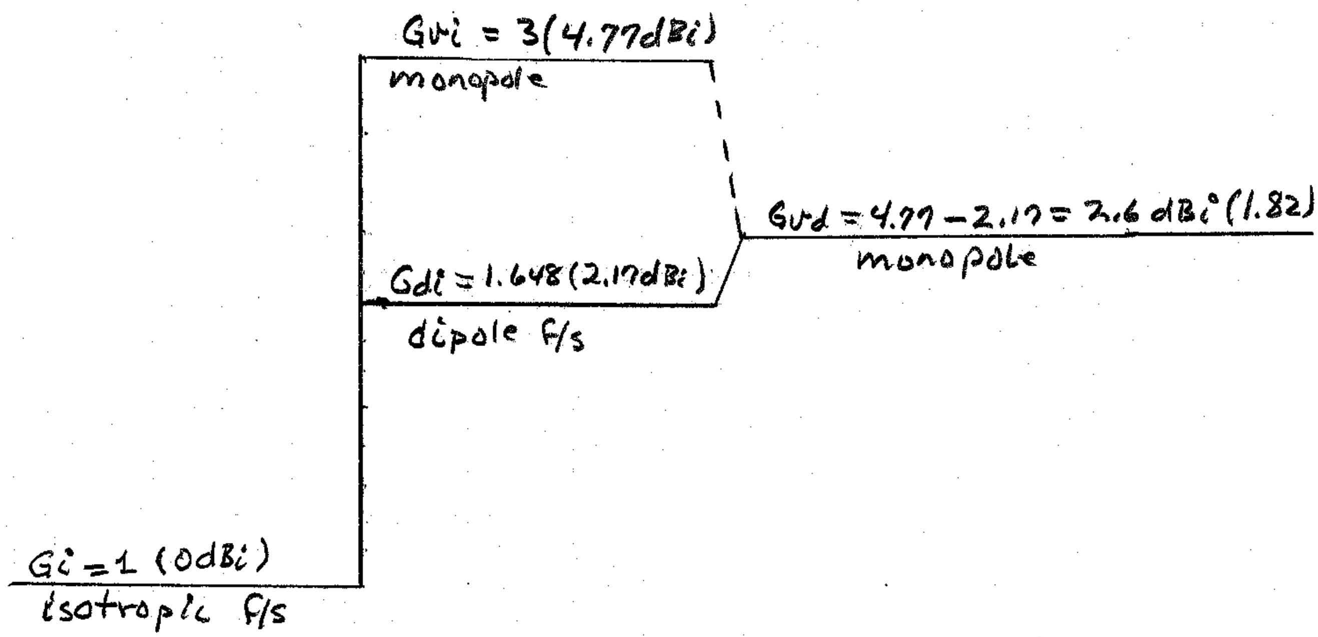

Figure SB2 -

From these relationships we can see that given an EIRP=5W, for a short vertical Pr=5/3=1.67W. Given an ERP=5W, for a short vertical Pr=5/1.82=2.75W.

Calculation of power density

To determine the power density (Pd) in the wave front we can make a field strength (|Ez|) measurement at some distance r from the antenna:

$$Pd = \frac{|Ez|^2}{377} \approx \frac{|Ez|^2}{120\pi} \quad \left[\frac{W}{m^2}\right] \quad (1)$$

Note, Ez is in V/m and 377 Ω represents the impedance of free space. Implicit in equation (1) is the assumption that the measurement of Ez has been taken far enough from the antenna to be in the far-field where |Ez|/|Hy|≈ 377 Ω. As shown in appendix C that condition does not exist until you are a considerable distance from the antenna. 1000m or 1 km is often cited as the desired distance for the measurement but as the discussion in appendix C shows that's really not far enough. At 630m you need to be at least five wavelengths away or about 3km, 5km would be better.

Assuming the Pd derived from an |Ez| measurement is constant over a sphere with radius r (in meters) you can multiply Pd by the area of the sphere to obtain EIRP:

$$EIRP = \frac{r^2|Ez|^2}{60} \quad [W]$$

The point of all this is that while we may be allowed an EIRP = 5W, the allowed Pr is about 1.7W!

References

[1] Carl Johnk, Engineering Electromagnetic Fields and Waves, John Wiley & Sons, 1975

[2] Stutzman and Thiele, Antenna Theory and Design, John Wiley & Sons, 1981

[3] Brown, Lewis and Epstein, “Ground Systems as a Factor in Antenna Efficiency,” *Proc. IRE* , Jun 1937, pp. 753-787

[4] R. Collin and F. Zucker, *Antenna Theory*, Chap 23 by J. Wait, Inter-University Electronics Series (New York: McGraw-Hill, 1969), Vol 7, pp 414-424.

[5] Leitner and Spence, Effect of a Circular Ground-plane on Antenna Radiation, Journal of Applied Physics, October 1950, Vo.. 21, pp. 1001-1006

[6] Hansen and Beckerley, "Concerning New Methods of Calculating Radiation Resistance, Either With or Without Ground", IRE proceedings, Vol. 24, No. 12, December 1936, pp. 1594-1621, see table III, page 1617

[7] Edmund Laport, Radio Antenna Engineering, McGraw-Hill, 1952